Design guide for memory®-steel

03.2022

Index of symbols

Latin letters

A_f	cross-sectional area of re-plate or re-bar
A_s	total cross-sectional area of reinforcement
a_s	reinforcement area per metre
b	width of concrete cross-section
d	effective depth of reinforcement
d_f	effective depth of re-plate or re-bar
E_c	elastic modulus of concrete
E_{SMA}	simplified elastic modulus of memory®-steel after activation
F_c	concrete compressive force
f_{cd}	design concrete strength
$F_{ms,u}$	memory®-steel tensile force for cross-sectional analysis
$F_{p,i}$	memory®-steel prestressing force directly after activation at t = 0
$F_{p,\infty}$	memory®-steel prestressing force after relaxation at t = ∞
F_s	tensile force in reinforcement cross-section
f	estimated maximum concrete slab/beam deflection according to [1]
h_c	thickness of concrete slab
I	moment of inertia
l	concrete slab/beam span
l_b	anchorage length
L	free length of re-plate between the anchorages
ΔL	length change of re-plate up to failure according to [1]
M_{Ed}	design bending moment
$M_{p,BZ}$	prestressing moment from memory®-steel in construction state
$M_{p,GZ}$	prestressing moment from memory®-steel after relaxation (for limit state calculation)
M_{Rd}	design value bending resistance
m_{Rd}	design value bending resistance of a concrete slab
P_0	prestressing force of a tendon at t = 0
P_∞	prestressing force of a tendon at t = ∞
V_{Ed}	design value shear force
V_{Rd}	design value shear resistance
$V_{Rd,s}$	shear resistance of re-plate end anchorage with Hilti X-CR nails
w_{eff}	existing deflection
w_{all}	allowable deflection
x	depth of bending compression zone
z	lever arm

Greek letters

ε_0	prestrain of a tendon
ε_c	concrete strain
ε_s	reinforcement steel strain
ε_f	memory®-steel strain
$\Delta\varepsilon_f$	memory®-steel strain increase due to length change
$\Delta\sigma_f$	stress increase in memory-steel
$\Delta\sigma_{p,r}$	prestress loss after relaxation (after 50 years)
σ_c	concrete stress
$\sigma_{p,i}$	initial memory®-steel prestressing directly after activation
$\sigma_{p,\infty}$	long-term memory®-steel prestressing after relaxation

Introduction

Designing with memory®-steel products follows the usual structural design rules for reinforced and prestressed concrete structures. The «re-plate» strengthening plate is considered an unbonded external strip with prestressing. A rigid bond between the installed ribbed steel and the surrounding mortar/sprayed concrete can be assumed for the «re-bar» system. Design proposals for the flexural strengthening of structures, in their serviceability and load capacity limit states, are explained below. For clarity of understanding, some examples are also then shown.

Theoretical design principles

re-plate

Structural condition:

At construction state, it is important to check for possible cracking on top of the slab due to prestressing. The initial memory®-steel prestressing $\boldsymbol{\sigma_{p,i}}$ is applied in this case. The prestressing can be set as a constant bending moment $\boldsymbol{M_{p,BZ}}$ between the anchorages, to be compared with the cracking moment.

$$M_{p,BZ} = F_{p,i} * z = \sigma_{p,i} * A_f * z \tag{1}$$

($\boldsymbol{A_f}$ = re-plate area, $\boldsymbol{z}$ = lever arm)

Serviceability limit state:

For the serviceability limit state over a long period, the initial prestressing $\boldsymbol{\sigma_{p,i}}$ must be reduced due to relaxation. Over a period of 50 years this can be estimated at 15%. The following equation applies:

$$\sigma_{p,\infty} = \sigma_{p,i} * \left(1 - \frac{\Delta\sigma_{p,r}}{\sigma_{p,i}}\right) \approx \sigma_{p,i} * 0.85 \tag{2}$$

The constant bending moment $\boldsymbol{M_{p,GZ}}$ between the anchorages can therefore be described as:

$$M_{p,GZ} = F_{p,\infty} * z = \sigma_{p,\infty} * A_f * z \tag{3}$$

Ultimate limit state:

In the re-plate system, the forces are transmitted to the structure through the two end anchorages; in the free length there is no bond with the concrete substrate. This means that a conventional cross-sectional analysis with strain compatibility is not possible. Two alternatives are possible:

a) Calculation without stress increase in re-plate:

This simplified calculation method assumes the tensile force $F_{ms,u}$ in the re-plate to be constant as the structural deformation increases. This assumption means that the force equilibrium in the cross-section is obtained by conventional cross-sectional analysis and the load capacity can be deduced. This calculation can be done manually, by data processing, e.g. Excel, or by computer software. This simplification is also used in standard design software with cross- sectional analyses.

$$F_{ms,u} = \sigma_{p,\infty} * A_f \tag{4}$$

This conservative assumption underestimates the actual load. The concept is suitable for cases where the serviceability limit state is critical for the structural design.

b) Calculation with stress increase in re-plate:

A second method is based on estimating the additional re-plate length change as the load increases, or slab deflection. The basis is an empirical design approach, obtained from loading tests on concrete beams with subsequent unbonded strand prestressing [1]. To summarise, based on the cross-section dimensions an additional maximum deflection f which causes a length change ΔL in the re-plate is estimated. The method assumes that all the deformation in a single-span beam is concentrated in a crack cross-section in the centre of the beam. This length change can be converted to additional re-plate elongation $\Delta\varepsilon_f$, which is limited to 0.7 % (from tests) and then gives the stress state $\sigma_{p,\infty} + \Delta\sigma$ in the lamella cross-section from the known stress-strain curve after activation. To simplify this, a reduced elastic modulus E_{SMA} of 70 GPa can be applied here to calculate the definitive strain through the change in elongation.

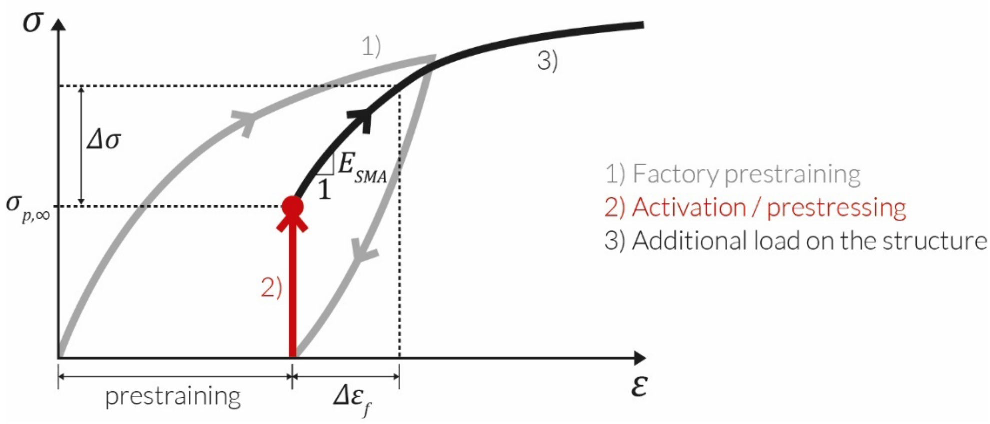


Figure 1: re-plate stress-strain diagram with pre-straining, activation, and subsequent loading

The following applies:

$$f = 0.9 * d - e_v < 0.02 * L \quad (5)$$

($\boldsymbol{d}$ = effective depth, $\boldsymbol{e_v}$ = 0 in the case of straight lamellas, $\boldsymbol{L}$ = free length of re-plate between the anchorages)

$$\Delta L = \frac{4 * f * z}{L} \quad (6)$$

$$\Delta \varepsilon_f = \frac{\Delta L}{L} < 0.7\ \% \quad (7)$$

Based on the known additional strain and therefore additional stress, the tensile force in the re-plate is also known and a force equilibrium in the cross-section can be calculated. Hence the maximum load capacity is determined. The specific national design principles for concrete structures (concrete compression and tensile failure of the reinforcement) apply, with adapted material parameters.

Anchorage:

The additional tensile force in the re-plate at ultimate load must be compared with the anchoring resistance.

$$F_{ms,u} = \left(\sigma_{p,\infty} + \Delta\sigma_f\right) * A_f \leq V_{Rd,s} = {}^{108\ kN}/_{1.3} = \mathbf{83.1\ kN} \quad (8)$$

Note: The anchoring resistance at ultimate load is the controlling value for standard structure's geometries. An explicit verification can be neglected in most cases. The anchoring resistance of 108 kN count for 12 Hilti X-CR nails and is reduced by a safety factor of 1.3 (recommendation re-fer). The specification applies for concretes with a measured compressive strength (cube) of >20 N/mm^2. The re-fer engineering support can be contacted for cases with lower concrete qualities.

re-bar

Structural condition:

Usually, re-bar is anchored bilaterally in the anchorage regions at both ends through a Sika mortar layer on the bearing substrate and the intermediate regions are prestressed. The load-bearing capacity is the same for the re-plate, as the exposed area acts as an external tie rod. By analogy, equation (1) can be applied to re-bar with the corresponding cross-sectional area.

Serviceability limit state:

After initial activation/prestressing of the re-bar, the regions between the anchorages are grouted, resulting in a firm bond with the load-carrying structure. Calculation can be done by conventional cross-sectional analysis with deduced elongation compatibility and force equilibrium. The initial prestressing $\boldsymbol{\sigma_{p,i}}$ must be reduced for the serviceability limit state, due to the relaxation under equation (2).

For calculations of deflection reduction due to prestressing, a homogeneous bending moment can again be assumed (see equation (3)), for example to solve the problem with the principle of virtual work.

Ultimate limit state:

The same principles of cross-sectional analysis apply to calculating the structural safety. Dependent on the situation, re-bar now has additional strain/stress added to the initial prestressing. The strain change in the re-bar consists of the additional strain between the application/prestressing date and the failure state ($\Delta\varepsilon_f$).

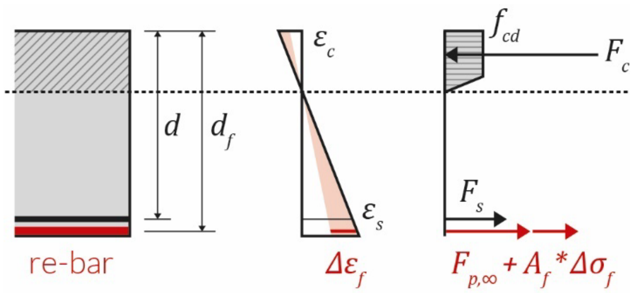


Figure 2: Schematic diagram for cross-sectional analysis of the ultimate limit state

The force equilibrium is then carried out with an equivalent force in the re-bar, which is made up as follows. For simplicity, a reduced elastic modulus E_{SMA} of 70 GPa can again be applied. The final force must be smaller as the maximum tensile force of re-bar.

$$F_{ms,u} = F_{p,\infty} + A_f * \Delta\sigma_f = A_f * (\sigma_{p,\infty} + \Delta\varepsilon_f * E_{SMA}) \qquad (9)$$

Anchorage:

The re-fer guidelines propose values for the embedded anchorage length of re-bar. The anchorage regions are dependent on the anticipated tensile forces, bar diameters and application (in cut grooves, cover concrete or sprayed concrete). Standard requirements for the adhesive strength, roughness etc. must also be met. Class R3 and R4 mortars from Sika according to EN regulations for concrete repairs are used for existing concrete load bearing substrates. A pull-off strength of the concrete substrate of 1.5 N/mm^2 is recommended.

Flexural strengthening must be anchored behind or on the zero-moment line. Transfer of the prestressing force to the load-bearing concrete substrate is normally achieved purely through the mortar bond. Alternatively, approved dowelling systems or other special solutions anchoring in the concrete compression zone may be used.

Notes

Specific product parameters should be taken from the current national product data sheets as required. Values used in design examples may vary from the current material parameters due to product and standards updates and should always be checked. The re-fer engineering support assists if anything is unclear and/or for specific design situations. For further information please visit our website: www.re-fer.eu (e.g. regarding our technologies, references, technical data sheets, tender texts, test reports etc.). Alternatively, please contact our Technical Support team directly for specific advice and assistance.

Corrosion

Appropriate measures should be taken in locations with chloride exposure and contamination, despite the good corrosion resistance of memory®-steel (risk of stress crack corrosion). Mortar cover on the re-bar should be re-assessed and increased if necessary. For re-plate products, a special coating is applied at the production facility (SikaCor® EG-1), which then limits the maximum heat temperature allowed to 165°C and therefore this also limits the maximum prestressing force.

Fire protection

Fire protection is always required for strengthening measures if the standard national fire load cannot be met without strengthening. The table below is a simple comparative example of the residual safety margins for fire protection on a load-bearing structure with «low» and alternatively «high» strengthening levels.

Load examples [kN/m^2]	**Before strengthening**	**After strengthening**	
		«low» strengthening level **+3.0**	«high» strengthening level **+5.0**
Dead load / applied load	5.0	5.0	5.0
Live load	3.0	3.0 **+ 3.0** = 6.0	3.0 **+ 5.0** = 8.0
Service load	**8.0**	**11.0**	**13.0**
Example with global safety factor	8.0 * 1.5 = 12.0	11.0 * 1.5 = 16.5	13.0 * 1.5 = 19.5
Load capacity to be covered	**12.0**	16.5	19.5
Fire protection Criterion: new working load must be < 12.0 (existing load capacity)	-	**11.0 < 12.0** Not required	**13.0 > 12.00** Required

If a «high» strengthening load level has to be reached, the retrofitting measure must also carry load in a fire scenario; a fire protection is then necessary for the strengthening product. The same regulations and standards apply to re-bar laid in concrete or cementitious mortar as for conventional steel reinforcement. A sprayed, cement-based fire protection mortar is normally used for re-plate (SikaCem® Pyrocoat).

Design examples

Flexural strengthening with re-plate

At the client's request, the structural walls (marked red) are to be removed to merge two existing rooms into one large living room. This change in the static system of the load-bearing structure would inevitably cause bending moment problems in the deck slab. The example below shows the flexural strengthening measure of the concrete slab. Other verifications such as the load transfer to the walls and lower floors, shear forces, punching issues etc., are not considered. The structural condition is not investigated.

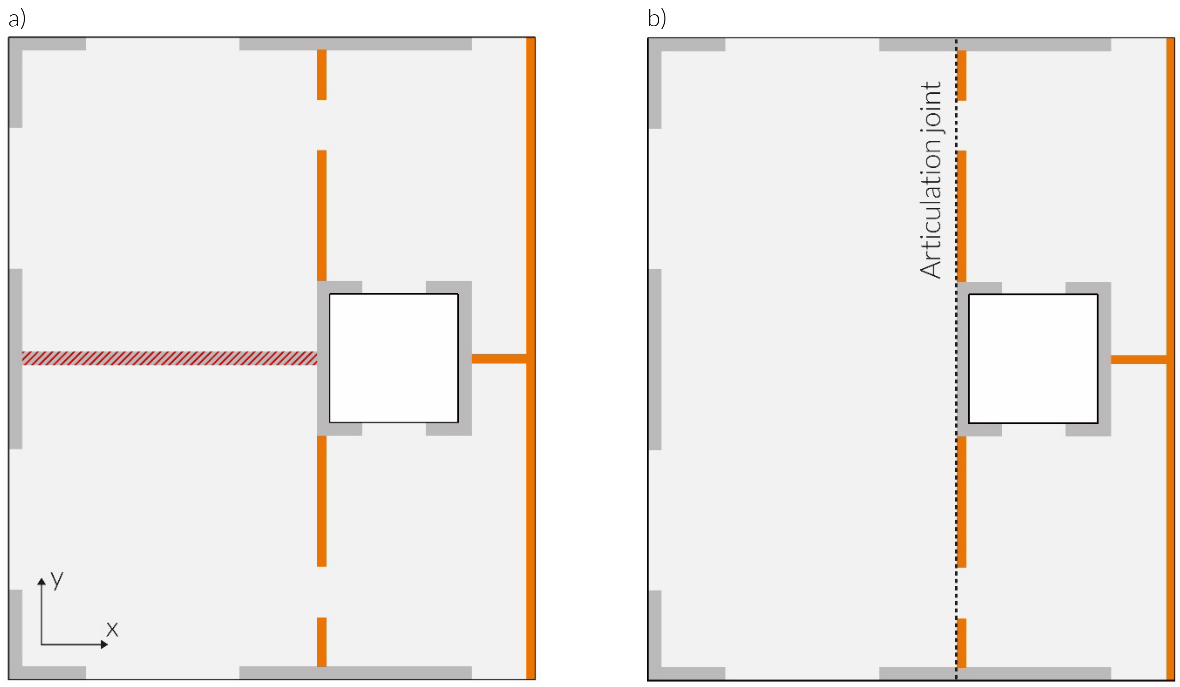


Figure 3: a) Model floor plan before, b) after removing the wall

In the existing slab, reinforcement of Ø10@150 (a_s = 524 mm^2 / m') was used and located in all layers. Concrete slab thickness is h_c = 200 mm, concrete quality C30 / 37 and the reinforcement cover 30 mm. R60 fire resistance is required for the load-bearing elements.

With these design data, bending resistance of m_{Rd} = 36 kNm/m' is obtained for the existing 1st/4th layers (x-direction). In the 2nd/3rd layers the bending resistance is 32 kNm/m'. With the new floor plan, the existing reinforcement (4th layer) is going to yield under the new permanent load. Therefore, an articulation joint is modelled in these regions to transfer that moment to the span (see Figure 3 b)).

Verification at service load level:

Under service loads, the new floor plan shows the following bending moments in x- and y-directions. At midspan in the main load-bearing direction, the bending resistance of the existing reinforcement is slightly exceeded. The strengthening system therefore must be fire protected. This is described in the Fire Protection section that follows.

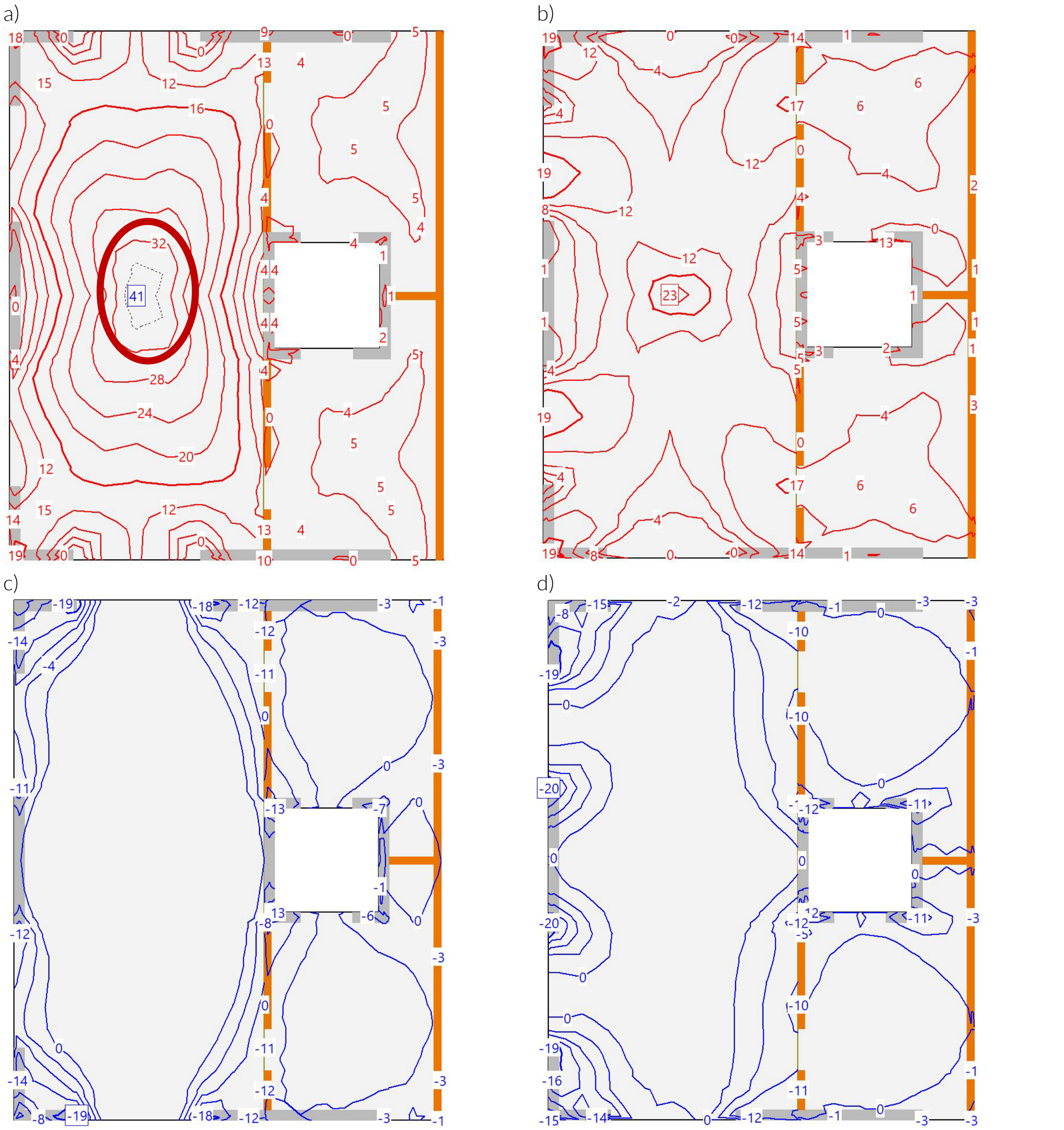


Figure 4: Plots of bending moments under permanent service load a) x-direction bottom (1st layer), b) y-direction bottom (2nd layer), c) x-direction top (4th layer), d) y-direction top (3rd layer)

A further factor in serviceability is deflection. Here, the cracked concrete cross-section under service load in the example has an effective deflection of 16.6 mm. The standards give the admissible value:

$$w_{all} \leq {}^{l}/_{300} = {}^{4'600\,mm}/_{300} = 15.3\,mm$$

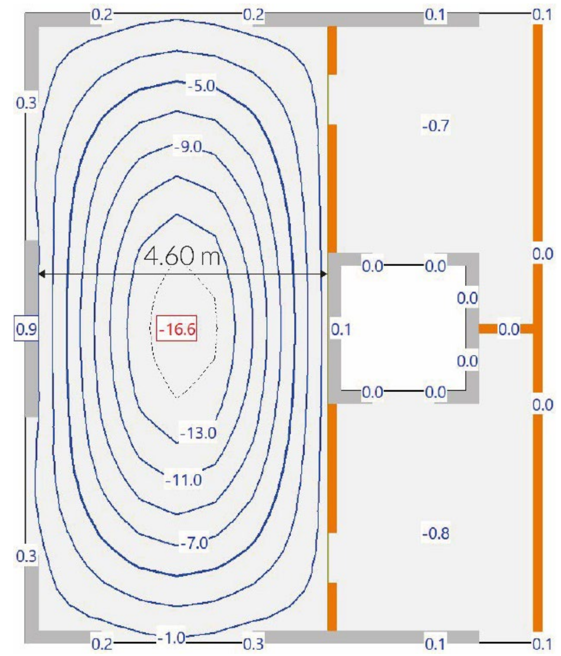


Figure 5: load deflection

Due to the prestressing, a constant moment can be applied at midspan across the approx. 3.0 m wide slab strip. Here, the established formula from the literature for a constant moment on a simple beam can be used. For more specialised cases (e.g. continuous beam), the design can be established by the principle of virtual work.

$$w = \frac{M * l^2}{8 * E_c I}$$

A simplified assumption is made that the whole concrete cross-section is cracked. This reduces the value $E_c I$ to $E_c I/3$. The actual equation then becomes:

$$w = w_{eff} - w_{all} = 16.6\,mm - 15.3\,mm = 1.3\,mm \leq \frac{M_{p,GZ} * l^2}{8 * (^{E_c I}/_3)}$$

That equation is solved according to $\boldsymbol{n}$ (the number of re-plate plates per metre):

$$w = \frac{M_{p,GZ} * l^2}{8 * (^{E_c I}/_3)} = \frac{\left(\sigma_{p,i} * 0.85 * A_f * z * \boldsymbol{n}\right) * l^2}{8 * \left(\frac{E_c * h_c^3 * b}{12 * 3}\right)}$$

$$\rightarrow \boldsymbol{n} = \frac{w * 8 * E_c * h_c^3 * b}{12 * 3 * \sigma_{p,i} * 0.85 * A_l * z * l^2}$$

$$= \frac{1.3\,mm * 8 * 33.6\,GPa * (200\,mm)^3 * 1.0\,m}{12 * 3 * 380 \frac{N}{mm^2} * 0.85 * 120\,mm * 1.5\,mm * \frac{200\,mm}{2} * (4.6\,m)^2} = \boldsymbol{0.63}$$

At least 0.63 re-plates per linear metre are required at the midspan. Unless the structural safety check indicates a larger figure, the strengthening plates therefore are applied every approx. 1.6 m.

Verification of structural safety at ultimate limit state:

The structural safety verification is carried out using the calculation with stress increase in the re-plate. The bending moments to be covered are shown as follows:

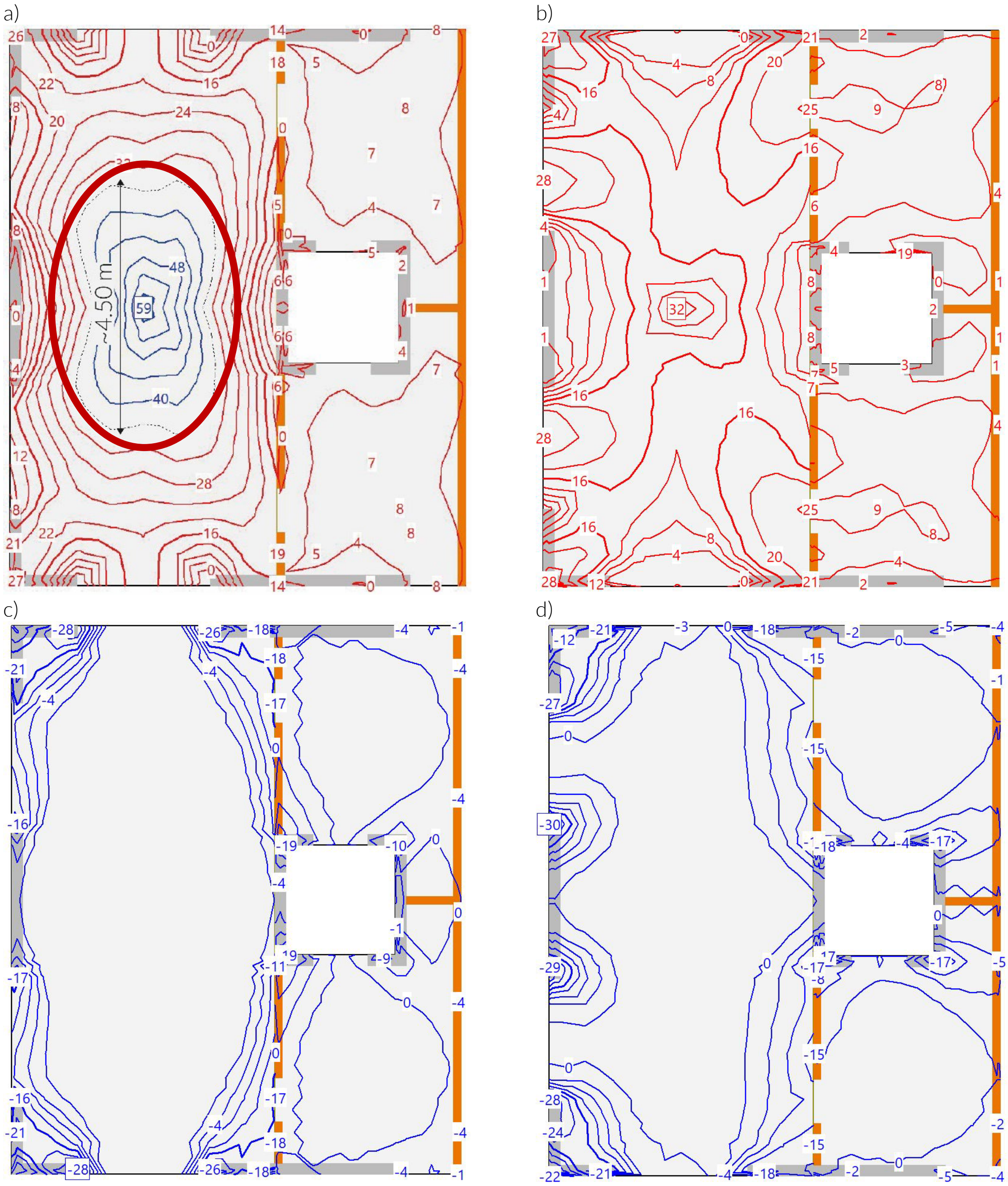


Figure 6: Plots of bending moments at structural safety ultimate limit state a) x-direction bottom (1st layer), b) y-direction bottom (2nd layer), c) x-direction top (4th layer), d) y-direction top (3rd layer)

To do this, firstly the elongation growth in the re-plate is calculated. The value for L (free length of re-plate between the anchorages) is obtained from the bilateral reduction in anchorage length (400 mm) and a safety margin (100 mm):

$$L = 4.6\, m - 2 * (400\, mm + 100\, mm) = 3.6\, m$$

$$f = 0.9 * d - e_v = 0.9 * (0.9 * 200\, mm) - 0 = 162\, mm < 0.02 * L = \boldsymbol{72\, mm}$$

$$\Delta\varepsilon_f = \frac{\Delta L}{L} = \frac{4 * f * z}{L^2} = \frac{4 * 72\, mm * (0.9 * 200\, mm)}{(3.6\, m)^2} = \mathbf{0.4}\% < 0.7\%$$

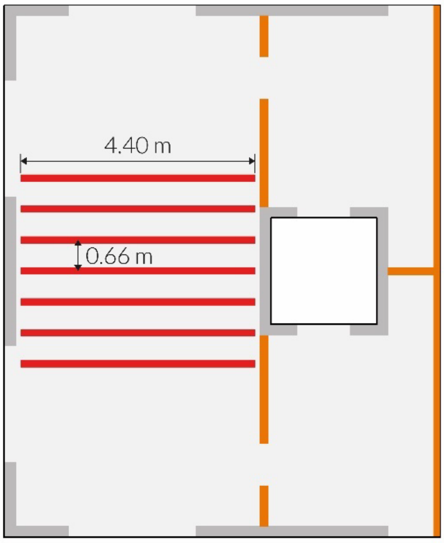
The bending moment for strengthening is therefore derived with the final force $F_{ms,u}$ in $\boldsymbol{n}$ re-plates through an internal lever arm $\boldsymbol{z}$ of about 0.9 * h_c:

$$F_{ms,u} = (\sigma_{p,\infty} + \Delta\sigma) * A_f = (\sigma_{p,i} * 0.85 + \Delta\varepsilon_f * E_{SMA}) * A_f$$
$$= (380 \frac{N}{mm^2} * 0.85 + 0.004 * 70\, GPa) * 120\, mm * 1.5 = 108.5\, kN < \mathbf{83.1}\, \boldsymbol{kN}$$

$$M_{p,GZ} = \boldsymbol{n} * F_{ms,u} * z = \boldsymbol{n} * 83.1\, kN * 0.9 * 200\, mm \geq 58.6\, kNm - 36.0\, kNm = 22.6\, kNm$$

$$\rightarrow \boldsymbol{n} = \frac{M_{p,GZ}}{F_{ms,u} * z} = \frac{22.6\, kNm}{83.1\, kN * 0.9 * 200\, mm} = \mathbf{1.5}$$

Hence, to cover the structural safety requirements in the overstressed regions (around 4.5 m), 1.5 re-plates are necessary per linear metre – e.g. one plate every 0.66 m (a total of 7).



7 x re-plate
l = 4.40 m, every 0.66 m,
with fire protection

Figure 7: Position of re-plate strengthening

Fire protection:

For fire, the quasi-permanent loads must be covered. As the flexural load capacity in the existing structure is insufficient, the strengthening measures are protected for R60. The sprayed, cement-based fire protection mortar SikaCem® Pyrocoat is used with a layer-thickness according to the current table in the re-plate data sheet.

Strengthening of a T-beam with re-bar

Due to a change of use and additional loads, various T-beams in a factory building need structural strengthening. This calculation example illustrates the method for excessive deflection in the main span and strengthening for flexural and shear problems in an individual beam of this kind. Additional verifications are omitted in this example. The beams covered two spans of 12.00 and 8.00 m and were simply supported.

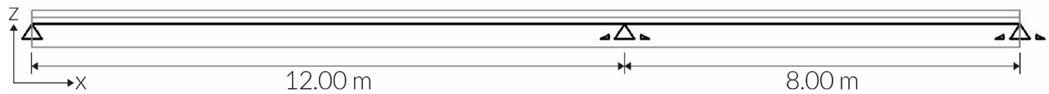


Figure 8: Two span beam in the factory building

The previous static forces (bending moments and shear forces) are shown below; there are no additional normal or torsion forces.

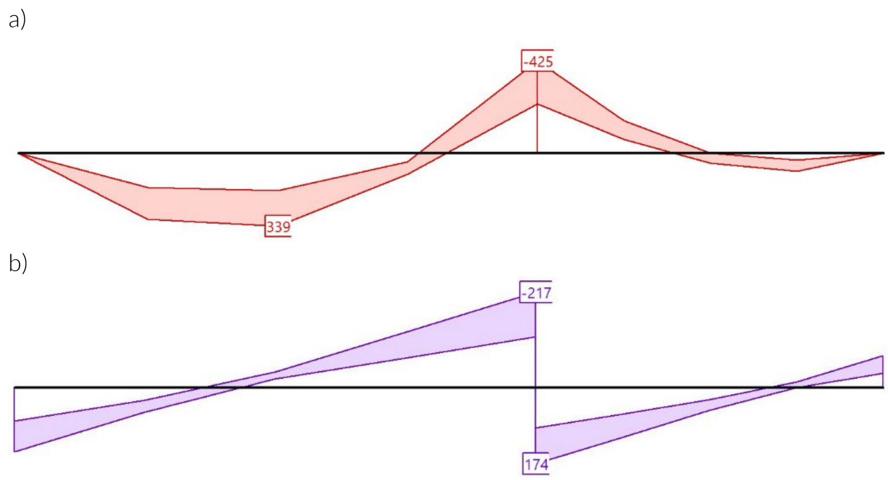


Figure 9: Internal forces at structural safety ultimate limit state a) Bending moment M_y [kNm] b) Shear loads V_z [kN]

In line with the original loadings, the beams were designed and reinforced as shown in Figure 10. The resultant deflection in the cracked concrete cross-section met the required standard specifications (w_{eff} = 32 mm / w_{all} = 34 mm).

Due to the client's new requirements, live loads are increased. A higher dead load also must be supported due to the additional mortar layer to be added. The resultant static forces for the structural safety ultimate limit state are as follows:

	Previous internal forces	Previous resistances	New internal forces
Bending moment [kNm]	M_{Ed} +339 -425	M_{Rd} +355 -440	M_{Ed} +449 -550
Shear force [kN]	V_{Ed} 217	V_{Rd} 230	V_{Ed} 285

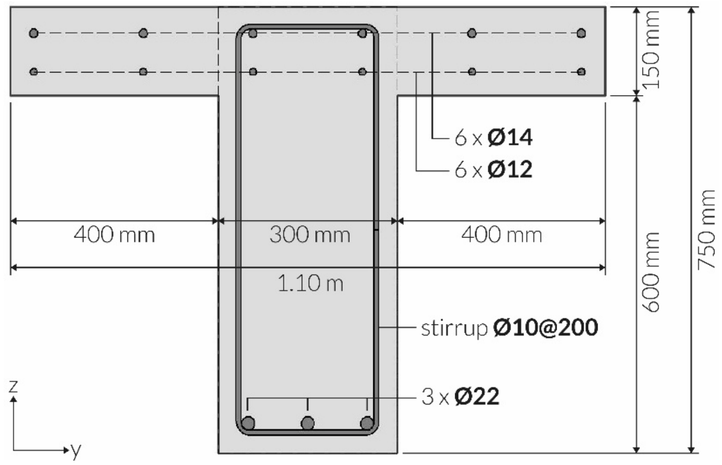


Figure 10: Existing cross-section of T-beams

Verification of structural safety at ultimate limit state:

Firstly, the structural safety ultimate limit state is investigated. The new internal forces are also shown in detail below.

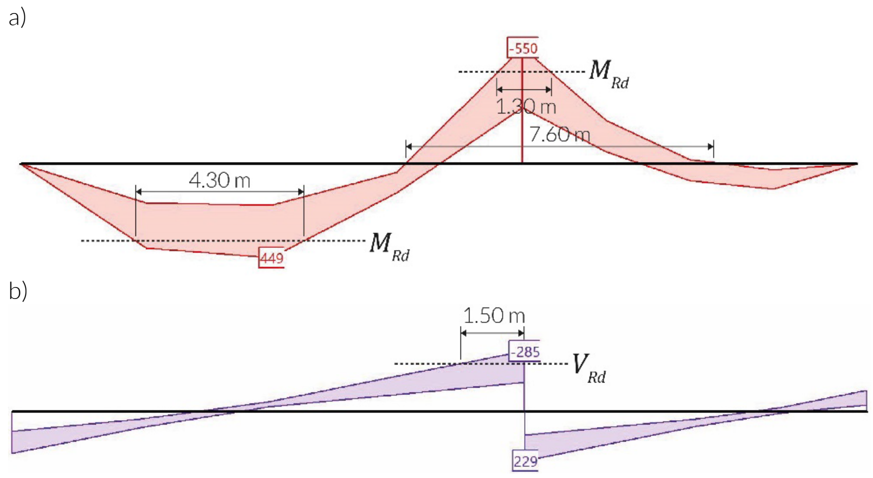


Figure 11: New internal forces at structural safety limit state a) Bending moment M_y [kNm] b) Shear forces V_z [kN]

Due to the additional loads, a shear problem occurs in a region about 1.5 m wide adjacent to the central point of support. The missing transverse shear strength of ca. 55 kN/m' is accommodated using re-bar 10 U-profiles. For simplicity, only the prestressing force (no strain increase up to shear failure) on the double shear stirrups is assumed.

$$V_{Rd,s} = \frac{2 * \sigma_{p,\infty} * A_f}{s} * z * \cot(45°) = \frac{2 * 350 \frac{N}{mm^2} * 0.85 * 89.9\, mm^2}{0.5\, m} * \sim 0.7 m * \cot(45°) = 75\, kN/m'$$

Accordingly, a total of three re-bar 10 U-profiles at a 0.5 m interval are necessary to strengthen the region. The stirrups are guided around the existing, roughened concrete surface and over the additional longitudinal re-bar. They are then embedded in sprayed mortar / grouted in the flange (anchorage over the neutral axis). The re-bar shear stirrups are electrically heated/activated from above. Spacers are installed to ensure that there is no contact with the existing reinforcement (electric tension loss during heating process).

In the larger sub-span, the new bending effect exceeds the previous resistance by some 94 kNm. Over the whole span, three re-bar 16 are installed on the bottom side of the web and embedded in sprayed mortar. Across the central support, the negative bending moment exceeds the permitted load over a length of ca. 1.3 by approx. 110 kNm. In that zone, four re-bars 10 are laid in fresh concrete cover (Note: anchorage of strengthening behind the zero-moment line). The strengthening bars are grouted in the anchorage region and heated after hardening, e.g. with a gas burner. Finally, the remaining zones are also embedded.

Flexural verification of the new cross-section can be done with standard design software. The new resistance levels are listed in the table below.

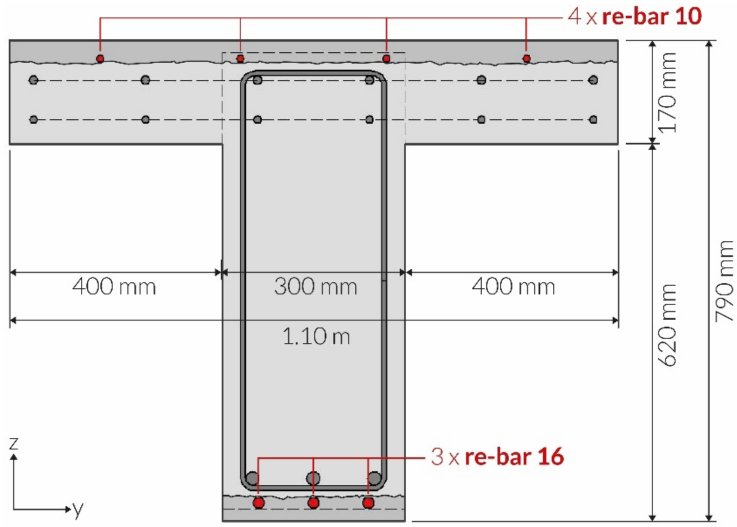


Figure 12: new cross-section of T-beams with re-bar flexural strengthening

	Previous internal forces	Previous resistance	New internal forces	New resistance
Bending moment [kNm]	M_{Ed} +339 / -425	M_{Rd} +355 / -440	M_{Ed} +449 / -550	M_{Rd} +569 / -553
Shear force [kN]	V_{Ed} 217	V_{Rd} 230	V_{Ed} 285	V_{Rd} 315

The following input parameters are used, amongst others, for the modelling:

Tendon attributes:

- Prestraining ε_0 = 0.57% for re-bar 10 and 0.46% for re-bar 16 (which gives theoretical prestressing respectively of: *Elastic modulus* * ε_0 = 400 N/mm^2, and 320 N/mm^2)
- Prestressing with bond
- Loss factor P_∞/P_0 = 0.85 (relaxation)

Material properties:

- *Elastic modulus* = 70 kN/mm^2 (re-bar elastic modulus after activation)
- $f_{p0.1k}$ = 520 N/mm^2 (Design value reduced by safety factor)
- ε_{ud} = 30%

Verification at service load level:

By installing prestressed strengthening elements embedded in mortar, crack openings are limited at the surface, and load is removed from the existing reinforcement. In addition to the improved durability, this example also investigates the deflection. Due to the new loads, the vertical deflection in the large span is calculated at about 39 mm. Flexural strengthening with three re-bar 16 implies a constant bending moment which counteracts the deflection. The resulting 5 mm (w_{eff} = 39 mm / w_{all} = 34 mm) should be eliminated with this measure.

The deformation of the statically indeterminate system implied by the prestressing can be calculated in various ways. Here, the principle of virtual work for the statically indeterminate system is used. As a basic system (BS), an articulated joint is introduced at the central support. For simplicity, the prestressing in the negative bending region is not included, though it would also have a positive effect.

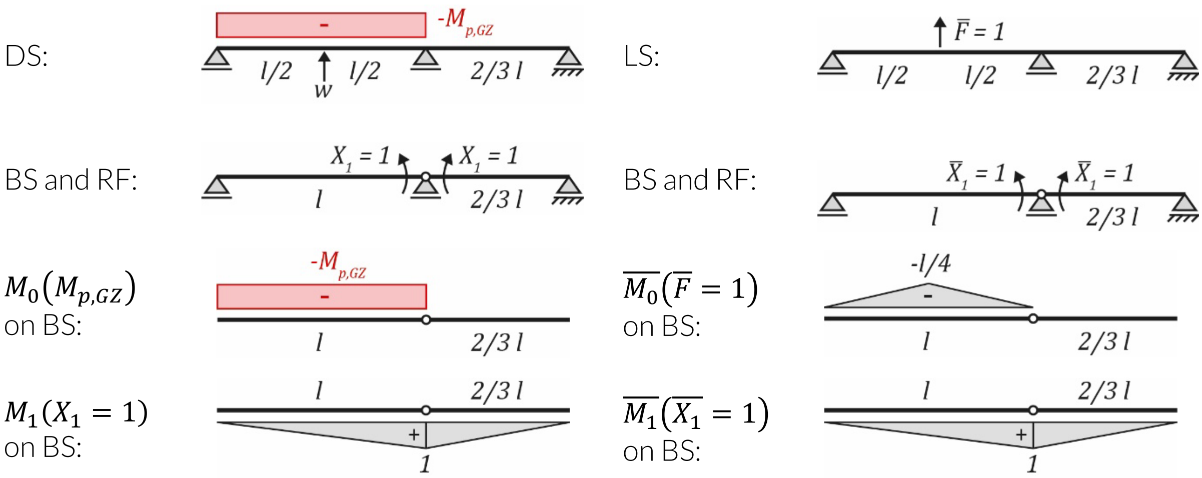


Figure 13: Simplification and reduction of the statically indeterminate system and principle of virtual work

$$\delta_{10} = \int M_1 * \frac{M_0}{E_c I} dx = \frac{1}{2} * (+1) * (-M_{p,GZ}) * \frac{l}{E_c I} + 0 = -\frac{M_{p,GZ} * l}{2 * E_c I}$$

$$\delta_{11} = \int M_1 * \frac{M_1}{E_c I} dx = \frac{1}{3} * (+1)^2 * \frac{\left(1 + \frac{2}{3}\right) l}{E_c I} = \frac{5 * l}{9 * E_c I}$$

$$\delta_{10} + X_1 * \delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{\mathbf{9}}{\mathbf{10}} \boldsymbol{M_{p,GZ}}$$

The deformation w can be deduced from this as follows:

$$w = \int \overline{M_0} * \frac{M_0}{E_c I} dx + X_1 * \int \overline{M_0} * \frac{M_1}{E_c I} dx$$
$$= \frac{1}{2} * \left(-\frac{l}{4}\right) * \left(-M_{p,GZ}\right) * \frac{l}{E_c I} + \left(\frac{9}{10} M_{p,GZ}\right) * \frac{1}{4} * \left(-\frac{l}{4}\right) * (+1) * \frac{l}{E_c I}$$
$$= \frac{M_{p,GZ} * l^2}{E_c I} * \left(\frac{1}{8} - \frac{9}{160}\right) = \frac{\mathbf{11 * M_{p,GZ} * l^2}}{\mathbf{160 * E_c I}}$$

Equation (3) gives the constant bending moment $M_{p,GZ}$ across the 12.00 m:

$$M_{p,GZ} = F_{p,\infty} * z = \sigma_{p,\infty} * A_f * z = 3 * 320 \frac{N}{mm^2} * 0.85 * 211.2\, mm^2 * \sim 0.66\, m = 114\, kNm$$

In addition, a reduced, cracked bending stiffness of the concrete cross-section is estimated ($E_c I_{cracked} = E_c I/3$) and included in the equation.

$$w = \frac{\mathbf{11 * M_{p,GZ} * l^2}}{\mathbf{160 * \left({}^{E_c I}/_{3}\right)}} = \frac{11 * 114\, kNm * (12.00\, m)^2}{160 * \frac{647'000\, kNm^2}{3}} = 5.2\, mm$$

The three re-bars installed to increase the structural safety consequently contribute to a reduction in the deflection of around 5 mm. The verification is achieved.

Verification of anchorage regions:

The negative and positive bending resistances were determined by a cross-sectional analysis software. The maximum tensile force in the re-bar and a tensile adhesion strength of the concrete of 1.5 N/mm^2 is used to design the anchoring zone. The resistance is reduced by a safety factor of 1.5. Four re-bar 10 are applied for the negative bending. Out of this, the following calculation for the necessary bond length l_b results:

$$F_{p,i}(negative) = 4 * \sigma_{p,i} * A_f = 4 * 520 \frac{N}{mm^2} * 89.9\, mm^2 = 187.0\, kN$$

$$F_{p,i} \leq \frac{l_b * 1.10\, m * 1.5 \frac{N}{mm^2}}{1.5} \rightarrow l_b = \mathbf{170\, mm}$$

The strengthening measure is embedded entirely in mortar. The anchorage region is assumed to be 300 mm of length.

In the case of strengthening against positive bending, three re-bar 16 are mounted on the bottom side of the web (width 300 mm). Again, the total maximum tensile force of the re-bars is anchored.

$$F_{p,i}(positive) = 3 * \sigma_{p,i} * A_f = 3 * 520 \frac{N}{mm^2} * 211.2\, mm^2 = 329.5\, kN$$

$$F_{p,i} \leq \frac{l_b * 300\, mm * 1.5 \frac{N}{mm^2}}{1.5} \rightarrow l_b = \mathbf{1'098\, mm}$$

This value can be optimized by using special solutions. As an example, the effect of the vertical prestressing by three re-bar U-profiles is presented. The tensile adhesion strength (1.5 N/mm^2) increases due to the vertical force of the prestressed U-profile in double shear (relaxation prestressing force 0.85 / safety factor 1.5).

$$F_{p,i} = 329.5\ kN \leq \frac{l_b * b * \left(1.5 \frac{N}{mm^2} + {}^{3*2*\sigma_{p,\infty}*A_f}/_{l_b*b}\right)}{1.5} =$$

$$\frac{l_b * 300\ mm * \left(1.5 \frac{N}{mm^2} + {}^{3*2*0.85*350\ N/mm^2*89.9\ mm^2}/_{l_b*300\ mm}\right)}{1.5} \rightarrow l_b = \mathbf{742\ mm}$$

Analogue to the intermediate support B, a shear strengthening with re-bar 10 U-profile is applied for support A, too. The anchorage zone is embedded in mortar over a length of 750 mm.

Schematic drawing:

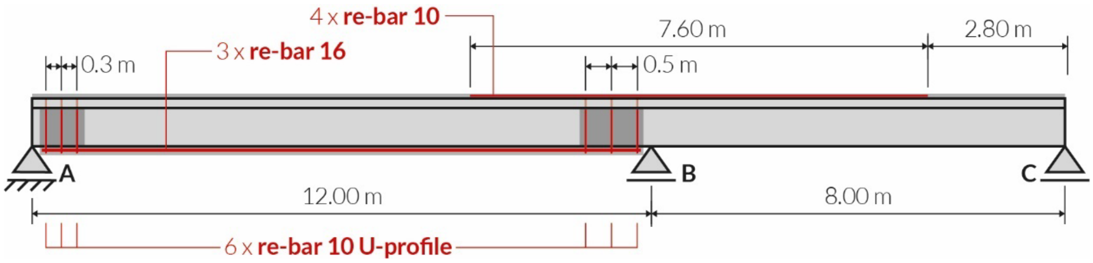


Figure 14: Sketch of strengthening works with re-bar longitudinal reinforcement and re-bar shear stirrups

The end regions of the re-bar flexural strengthening could also be made by conventional, slack-applied stirrups (steel B500B).

References

[1] Bruggeling, A.S.G., Voorspanning zonder aanhechting, enkelstrengsystemen. 1976, Delft Technical University: Delft, Netherlands.